

Notes on Sup-lattices

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1 Monoidal categories

These notes are based on the book [JohnsonYau2021].

1. A *monoidal category* is $(\mathbb{A}, \otimes, \alpha, \lambda, \rho, \mathbb{1})$, where:

- $\mathbb{A} \times \mathbb{A} \xrightarrow{\otimes} \mathbb{A}$ is a bifunctor, called the *monoidal product*,
- an object $\mathbb{1}$ of \mathbb{A} , called the *monoidal unit*,
- three natural isomorphisms — the *left unit isomorphism* $\langle \mathbb{1} \otimes X \xrightarrow{\lambda_X} X : X \in \mathbb{A}_0 \rangle$, the *right unit isomorphism* $\langle X \otimes \mathbb{1} \xrightarrow{\rho_X} X : X \in \mathbb{A}_0 \rangle$ and the *associativity isomorphism* $\langle (X \otimes Y) \otimes Z \xrightarrow{\alpha_{X,Y,Z}} X \otimes (Y \otimes Z) : X, Y, Z \in \mathbb{A}_0 \rangle$,
- these are subject to:

$$\begin{array}{ccc}
 ((X \otimes Y) \otimes Z) \otimes W & \xrightarrow{\alpha_{X,Y,Z} \otimes \mathbb{1}_W} & (X \otimes (Y \otimes Z)) \otimes W \\
 \alpha_{X \otimes Y, Z, W} \downarrow & & \downarrow \alpha_{X, Y \otimes Z, W} \\
 (X \otimes Y) \otimes (Z \otimes W) & \xrightarrow{\alpha_{X,Y,Z \otimes W}} X \otimes (Y \otimes (Z \otimes W)) & \xleftarrow{\mathbb{1}_X \otimes \alpha_{Y,Z,W}} X \otimes ((Y \otimes Z) \otimes W)
 \end{array}$$

and

$$\begin{array}{ccc}
 (X \otimes \mathbb{1}) \otimes Y & \xrightarrow{\alpha_{X, \mathbb{1}, Y}} & X \otimes (\mathbb{1} \otimes Y) \\
 \rho_X \otimes \mathbb{1}_Y \searrow & & \swarrow \mathbb{1}_X \otimes \lambda_Y \\
 & X \otimes Y &
 \end{array}$$

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