

How to choose a unit system for Poiseuille-Flow in Espresso

1. Start with a unit of length. A resolution of 50 lattice points over the 50 μm channel would be appropriate, therefore

$$[x] = 1 \mu\text{m}. \quad (1)$$

2. The lattice-Boltzmann algorithm only produces valid results within a certain parameter range. By choosing the parameters to be around 1, you should be on the safe side. In a system like this, we actually have enough freedom to scale the units, so that all parameters become 1 exactly. Doing this for the density yields the mass unit $[m]$

$$\begin{aligned} \rho &= 1 \times 10^3 \text{ kg m}^{-3} \stackrel{!}{=} 1 [m] [x]^{-3}. \\ \Rightarrow [m] &= 10^{-15} \text{ kg}. \end{aligned} \quad (2)$$

3. You get the unit for time $[t]$ by fixing the viscosity

$$\begin{aligned} \eta &= 1 \text{ mPa s}^{-1} = 1 \times 10^{-3} \text{ kg s}^{-1} \text{ m}^{-1} \stackrel{!}{=} 1 [m] [s]^{-1} [t]^{-1}. \\ \Rightarrow [t] &= 1 \mu\text{s}. \end{aligned} \quad (3)$$