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## Cryptanalysis on Graphic Cards or other Highly Parallel Architectures

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**112101115185:** 100-2628-E-001-004-MY3

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## Abstract

In this project we conducted cryptanalytic studies on parallel architectures. This is our specialty. We also made attacks on discrete logarithms and RSA earlier and made a name in the crypto community. This is a continuation of the study extending to the cryptanalysis of post-quantum public-key cryptography, which is a common research interest for us and for our long-term research partners, the CASED (Center for Advanced Security rEsearch, Darmstadt) team at TU Darmstadt headed by Academician Johannes Buchmann (of the National Academy of Sciences of Germany).

Post-Quantum Cryptography studies public-key cryptographic schemes which will not be dealt a fatal blow when large quantum computers appear. However, no one knows whether any of these can be made practically safe under conventional attacks. We seek to address this question.

In our major result we showed that multivariate cryptography can be assisted by highly parallel structures. We build a highly parallel XL solver which can be better than the best F4 and F5 algorithms previously considered champions. The program scales well to many cores or machines with high speed interconnect, and will be donated to open source project SAGE.

Our recent works were selected to CHES 2012, which is one of the biggest and most important conferences in cryptography, and something to be proud of. We have also some more minor results.

**Keywords:** GPU, parallelization, cryptanalysis, post-quantum cryptography

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# 1 11620980216186

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186Johannes Buchmann 2081942042412372391761192342361991240861241046761862081647721820464  
211661192321861121011137670119234234236124(DFC) cryptosystems.

186102214, 253792182041118317981214227,

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93116239211242243184(Lattice)80104220184(Multivariate)186225113108758819918624018767  
computer (cost: US\$600)

## 2 17871(Results)

PQCrypto (2251131087588199) means implementing cryptography that will survive the invention of quantum computers. We achieve breakthroughs in getting faster cryptanalysis and a better understanding of provably secure complexity offered for cryptosystems based on these systems.

### XL algorithm to solve systems.:

Solving a system of multivariate quadratic equations (MQ) is a hard problem whose complexity estimates are relevant to many cryptographic scenarios. In some cases it is required in the best known attack; sometimes it is a generic attack (such as for the multivariate PKCs), and some of the time it determines a provable level of security (such as for the QUAD stream ciphers).

Under some reasonable assumptions, the best way to solve generic MQ systems is the XL algorithm implemented with a sparse matrix solver such as Wiedemann. Knowing how fast one can implement this attack gives us a good idea of how

future cryptosystems related to MQ can be broken, similar to how implementations of General Number Field Sieve that factors smaller RSA numbers gives us more insight into the security of actual RSA-based cryptosystems.

This paper describes such an implementation of XL with Block Wiedemann. We are able to solve in 2.5 days, on a 48-core, 64GB RAM computer (cost: US\$600), a system with 30 variables and 60 equations over  $GF(16)$  (a computation of about  $2^{57}$   $GF(16)$ -multiplications). We do not expect  $F_4/F_5$  to accomplish this due to its much higher space usage. We are also able to solve in 1.3 days, on a single Amazon EC2 cc2.8xlarge instance, a system with 23 variables and 32 equations of  $GF(16)$ . This directly translates to an estimate of the cost to break the HFE Challenge 2 at  $US\$ < 2^{43}$ . The software can be easily adapted to other small fields including  $GF(2)$ . More importantly, it scales nicely for small clusters, NUMA machines, and a combination of both to test systems which are only 1.5 orders of magnitude away in bit-complexity from the Block Wiedemann used for RSA-768.

### The Comparison of XL vs $F_4/F_5$ vs Brute Force

We address the much needed comparison of XL vs vs  $F_4/F_5$  vs Brute Force and show that XL is better asymptotically in most cryptologically interesting situation than  $F_4/F_5$  by showing that the difference in the XL degree and the  $F_4/F_5$  degree of operation is often at most 1 asymptotically. This paper is in

preparation.

### **Brute-force attack using FPGAs vs GPUs**

We show that the CHES 2010 result can be extended to energy efficient FPGAs such that it can have much much better energy performance, even parts-cost-performance than GPUs, which we do in this (submitted) paper. We have to do a lot of engineering to get around obstacles posed by the Xilinx hardware and software tools. We are able to improve the energy to performance ratio by a factor of more than 10.

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## 4 List of Attachments

25424317698187112101245230186215229,

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- M.-S. Chen, C.-M. Cheng, B.-Y. Yang, RAIDq: A software-friendly, multiple-parity RAID, USENIX HotStorage 2013 (USENIX Federated Workshops, June 27-28, San Jose, CA, USA), to appear.
- S. Tanaka, T. Chou, B.-Y. Yang, C.-M. Cheng, K. Sakurai: Efficient Parallel Evaluation of Multivariate Quadratic Polynomials on GPUs, WISA 2012 (13th Workshop on Information Security Applications, August 16-18, Jeju Island, Korea), LNCS 7690, pp. 28-42.
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